Research and Analysis on Markowitz Model of Portfolio Selection

Mengyuan Zhang*

College of Art & Science, Boston College, 02467, Boston, United States

*Corresponding author: zhangcau@bc.edu

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Abstract: In the past few years, people focus more on portfolio selection and stock investment. It is increasingly significant to help investors to maximize their returns while minimizing the associated risks. In this paper, Full Markowitz Model is used as a basis to analyze and build portfolios. The dataset includes a recent 20 years of historical daily total return data from ten stocks. All optimal optimization inputs are calculated based on the monthly observations, including efficient frontier, minimal risk portfolio, optimal portfolio, and minimum portfolios frontier. Also, the optimal portfolios are also built based on the five cases of the additional constraints in real life. These optimization results help investors to choose their portfolios according to their needs.

1. Introduction

Nowadays, more people choose to invest ahead of time because investing is an effective way to build potential wealth in the future. An investment portfolio is a collection of assets that include investments like stocks, bonds, mutual funds, and exchange-traded funds. Generally, there is a trade-off between risk and returns. People who risk more in investing for a higher return may also suffer a large loss as well. Therefore, it is necessary to help investors build optimal investment portfolios that maximize their returns or minimize their risks.

Modern Portfolio Theory is mainly comprised of Markowitz's Portfolio theory introduced in 1952 and William Sharpe's contributions to the theory of financial asset price formation in 1964, which is also known as "CAPM" [1]. In 1990, Harry Markowitz received a Nobel Prize for his theoretical contributions to financial economics and corporate finance. His essay "Portfolio Selection: Effective Diversification" [2] has received a lot of attention from the public. Sharpe advanced the Efficient Frontier and Capital Market Line after his derivation of CAPM in 1964 [3]. Markowitz demonstrated that an investor's portfolio selection can be reduced to balance between two dimensions: the expected return of the portfolio and the risk or variance of the portfolio. Overall, the risk component of the Full Markowitz model can be reduced through diversification. As Markowitz mentioned, the core concept of the theory relies on the conventional wisdom of "never putting all your eggs in one basket" [4]. According to Frantz & Payne [5], the variance of a portfolio decreases as the number of assets in the portfolio increases, which significantly improves its efficient frontier. They find that the portfolio variance return of these assets is smaller than the weighted average of the individual asset variances [5]. Therefore, investors can reduce their risks by maintaining portfolios that are comprised of a larger number of assets.

However, it is important to note that "a truly diversified portfolio can often improve returns and significantly reduce unsystematic risk" [6]. The risk of a security can be divided into two components: systematic risk and unsystematic risk. Systematic risk is led by macroeconomic factors in the market including interest rate changes and foreign exchange fluctuations, which "affects a large number of assets to one degree or another" [7]. Unsystematic risk is caused by microeconomic factors that "specifically affect a single asset or narrow group of assets" [7].

Different from the traditional theory that analyzes the characteristics of individual assets, the statistical correlation of returns is an important component of the Full Markowitz Model because it affects the level of risk reduction [8]. The Full Markowitz Model focuses on the correlation between the assets and their weights in the portfolios [9]. Height points out that a portfolio with smaller correlation coefficient values suggests less risk [10], In other words, as the proportion of uncorrelated assets in the portfolio increases, the risk will be reduced at a greater level. In this case, the risk of a portfolio with a correlation coefficient of -1.00 can be fully eliminated.

This paper used Full Markowitz Model as a basis to build and analyze optimal investment portfolios. The dataset includes a recent 20 years of historical daily total return data from ten stocks, which belong in groups to three-four different sectors, one equity index, and a proxy for the risk-free rate. During the data procession, the daily data is transformed into the monthly data in order to reduce the non-Gaussian effects. The project is based on Full Markowitz Model method and follows the assumption of the model. All optimal optimization inputs are calculated based on the monthly observations, including efficient frontier, minimal risk portfolio, optimal portfolio, and minimum portfolios frontier. Also, the optimal portfolios are also built based on the five cases of the additional constraints in real life. These optimization results help investors to choose their portfolios according to their needs.

The remainder of the paper is organized as follows: Section 2 describes the sample and data; Section 3 introduces the methods in portfolio constructions, including the Full Markowitz Model and five additional constraints; Section 4 shows the allocation results and analysis, along with different optimal portfolios and weights; Section 5 presents the conclusions.

2. Data

In this paper, I collected a recent 20 years (from 2001 to 2021) of historical daily total return data for ten stocks from Yahoo!Finance, one (S&P 500) equity index (a total of eleven risky assets), and a proxy for risk-free rate (1-month Fed Funds rate). The ten stocks are companies from different sectors, including Adobe, IBM, SAP, BOA, C, WFC, TRV, LUV, ALK, and HA.

2.1. Adobe

Adobe Inc. was leading in the creation of programming language and developed Photoshop and PDF readers, which allow users to communicate information across all electronic media in more convenient ways. Its stock price experienced a rocket growth during the last half of 2021.

Figure 1. Adobe’s Stock Price Trend

2.2. IBM

International Business Machines (IBM) is an American multinational technology company selling computer hardware, software, and middleware. It is also a major research organization that holds lots of patents including ATM machines. Its stock price rose from 120 to 145 in the first half of the year but dropped backed to 120 recently.

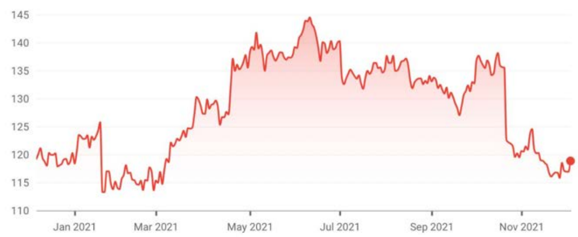

Figure 2. IBM’s Stock Price Trend

2.3. SAP

SAP SE is a German multinational software corporation started as a private partnership. As the largest non-American software company by revenue, it develops enterprise software to manage business operations and customer relationships. Its stock price climbed up to 150 in the second half of the year but dropped to 130 in November.

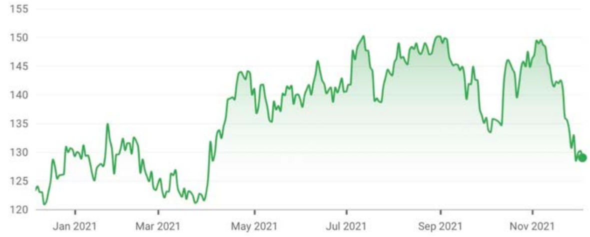

Figure 3. SAP's Stock Price Trend

2.4. BAC

As one of the Big Four banking institutions of the United States, the Bank of America Corporation is an American multinational investment bank and financial services holding company. Its operation includes several segments: Consumers Banking, Global Banking, Global Markets, and Global Wealth and Investment Management. The stock price increased steadily from 30 to 45 during the year.

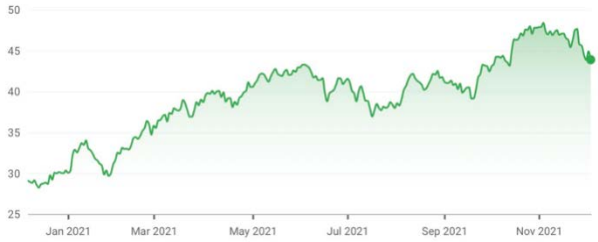

Figure 4. BAC's Stock Price Trend

2.5. C

As one of the Big Four banking institutions of the United States, Citigroup is also an American multinational investment bank and financial services corporation. It is considered as systematically important by the Financial Stability Board and cited as being too big to fail. The stock share went up during the first half of the year and kelp dropped later, decreasing to almost 63 in November.

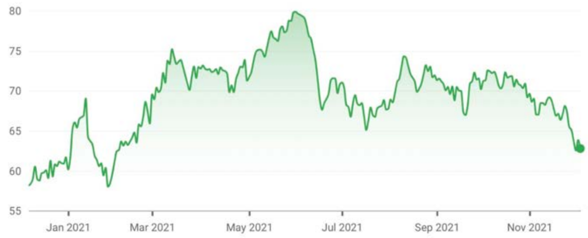

Figure 5. C's Stock Price Trend

2.6. WFC

Wells Fargo & Company is an American multinational financial service company and owns operations in 35 countries with over 70 consumers throughout the world. It is also considered as a systematically significant financial institution by the Financial Stability Board. Its stock share increased steadily from 30 to 48 during the year.

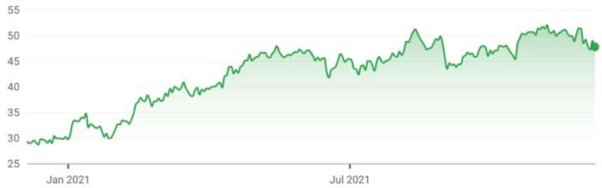

Figure 6. WFC's Stock Price Trend

2.7. TRV

The Travelers Companies, Inc is an American insurance company, which is also the second-largest writer of commercial property casual insurance in the United States. The stock price kept swinging but increased from 135 to 150 during the year.

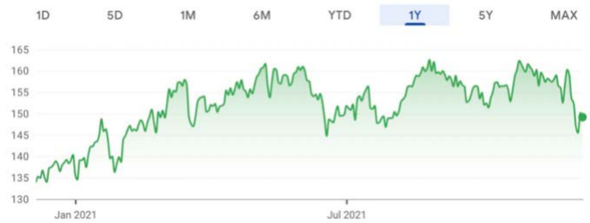

Figure 2. TRV's Stock Price Trend

2.8. LUV

Southwest Airlines Co. is one of the major airlines of the United States and the world's largest low-cost carrier. The stock share kept increasing from 47 to 65 during the first half of the year but kept dropping to 45 in November.

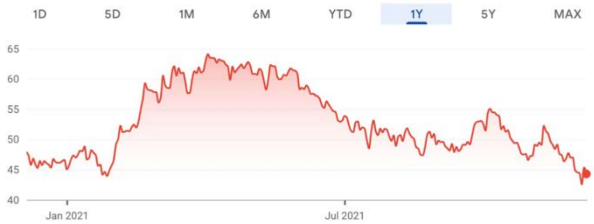

Figure 8. LUV's Stock Price Trend

2.9. ALK

Alaska Air Group is an airline holding company that owns Alaska Airline as a mainline carrier and Horizon Air as a regional carrier. Its stock price kept increasing from 50 to 75 during the first two quarters while kept decreasing during the second half of the year and dropped back to 50.

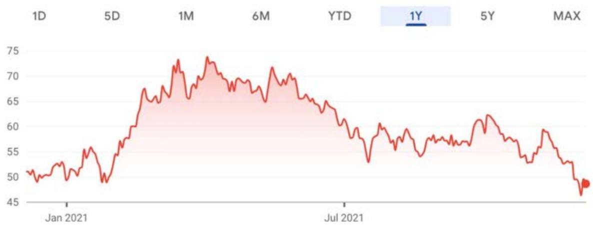

Figure 9. ALK's Stock Price Trend

2.10. HA

As the ten-largest commercial airline in the United States, Hawaiian Holdings, Inc. is the largest operator of commercial flights on Hawaii Islands. It also provides scheduled and charter air transportation of passengers, cargo, and mail. The stock share also experienced a substantial increase during the first half of the year while kept dropping later. The stock price dropped to 18 in November, which is even lower than the start of the year.

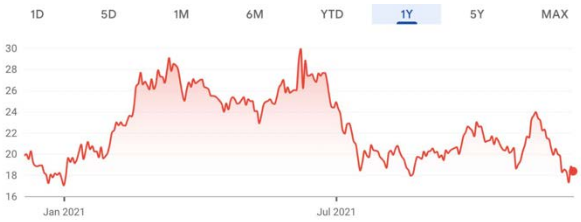

Figure 10. HA's Stock Price Trend

3. Method

In the project, Full Markowitz Model (MM) was used to explore the optimal portfolio of ten stocks under different constraints. The results will be compared, which is beneficial for investors to make decision analysis.

3.1 Full Markowitz Model

Full Markowitz Model, introduced by Harry M. Markowitz, is a theoretical framework for risk and return analysis. The statistical analysis aims to find the set of efficient portfolios that are expected to yield the highest return given the lowest risk possible. In addition, the diversification goals include (a) reducing the risk of security by reducing its volatility (variance or standard deviation); (b) reducing the covariance or interactive risk of securities in the portfolio

The Portfolio Theory of Markowitz is based on the following assumptions:

(1) The markets are efficient.

(2) Investors are free to access fair and correct information in the market.

(3) Investors are rational and try to maximize their utility.

(4) Investors are risk-averse and seek to maximize returns while minimizing risks.

(5) Investors make decisions on expected returns and variance or standard deviation of the returns for the mean.

(6) Investors always prefer higher returns to lower returns given certain levels of risks.

The formulas of Full Markowitz Model are shown as below:

(1) Portfolio return is the product of the vector of instruments' average returns and the unknown vector of instruments' weights:

$$r_p = \vec{w} \cdot \vec{\mu}^T \tag{1}$$

Where

$$\vec{\mu} = \{\mu_1, \mu_2, \dots, \mu_n\}^T \tag{2}$$

$$\vec{w} = \{w_1, w_2, \dots, w_n\}^T \tag{3}$$

(2) Portfolio standard deviation is the square root of the product of an auxiliary vector constituting of weights and standard deviations as well as a matrix of instruments' cross-correlation coefficient

$$\sigma_p = \sqrt{\vec{v} P \vec{v}^T} \tag{4}$$

Where

$$\vec{v} = \{w_1\sigma_1, w_2\sigma_2, \dots, w_n\sigma_n\}^T \tag{5}$$

$$P = \begin{Bmatrix} \rho_{11} & \rho_{12} & \cdots & \rho_{1n} \\ \rho_{21} & \rho_{22} & \cdots & \rho_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \rho_{n1} & \rho_{n2} & \cdots & \rho_{nn} \end{Bmatrix} \tag{6}$$

3.2 Constraints

There are five constraints designed for the optimization of portfolio, which are mostly reality factors including regulations and policies.

3.2.1. Constraint 1

This constraint is designed to simulate Regulation T by FINRA. This rule explains the margin requirements for fixed income securities, options, and equities. The regulation lets broker-dealers to allow their customers to have positions, 50% or more of which are funded by the customer's account equity:

$$\sum_{i=1}^{11} |w_i| \leq 2 \tag{7}$$

3.2.2. Constraint 2

This constraint is designed to simulate some arbitrary "box" constraints on weights of the portfolio, which sometimes are provided by the client.

$$|w_i| \leq 1, for\ \forall i \tag{8}$$

3.2.3. Constraint 3

This condition requires no additional optimization constraint. Under this condition, the optimization illustrates how the area of the permissible portfolio in general and the efficient frontier, in particular, look like when it is "free."

3.2.4. Constraint 4

This constraint is designed to simulate the typical limitations existing in the U.S. open-ended mutual fund industry which do not allow to have any short positions.

$$|w_i| \geq 0, for\ \forall i \tag{9}$$

3.2.5. Constraint 5

This constraint considers the case when the broad index is included in the portfolio and checks whether it will have a positive or negative effect. In this case, an additional constraint is added:

$$w_1 = 0 \tag{11}$$

4. Result Analysis

First, based on the Full Markowitz Model, the portfolios are all optimized under five constraints. Table X summarizes the weights of all optimization portfolios under each constraint. It is noticeable that under the condition of minimizing variance, most of the weights of the optimal portfolios are allocated into WFC and TRV. In this case, investors may choose to short ADBE, SAP, C, LUV, ALK, AND HA. Besides, under the condition of maximizing sharp ratio, most of the weights are allocated into ADBE, BAC, WFC, TRV, ALK, and HA while investors should consider short IBM, C, and LUV. Therefore, investors should choose the best optimal portfolio that meets their expectations and make a trade-off between risks and returns.

Table 1. Result Table for Weights

MM (Constr1):	SPX	ADBE	IBM	SAP	BAC	C	WFC	TRV	LUV	ALK	HA
MinVar	110.97%	-9.68%	5.14%	-9.90%	0.35%	-22.54%	14.06%	19.45%	-0.13%	-4.86%	-2.85%
MaxSharpe	39.2%	28.2%	-0.1%	0.5%	16.7%	-48.1%	19.8%	30.2%	-1.8%	7.7%	7.6%
MM (Constr2):	**SPX**	**ADBE**	**IBM**	**SAP**	**BAC**	**C**	**WFC**	**TRV**	**LUV**	**ALK**	**HA**
MinVar	100.00%	-8.53%	8.25%	-9.24%	0.66%	-23.03%	16.19%	22.93%	0.28%	-4.76%	-2.75%
MaxSharpe	50.09%	35.81%	-22.32%	7.19%	29.74%	-66.14%	20.56%	35.57%	-14.83%	13.45%	10.87%
MM (Constr3):	**SPX**	**ADBE**	**IBM**	**SAP**	**BAC**	**C**	**WFC**	**TRV**	**LUV**	**ALK**	**HA**
MinVar	111.46%	-9.91%	5.27%	-10.07%	0.60%	-22.81%	14.06%	19.53%	-0.26%	-4.98%	-2.88%
MaxSharpe	50.09%	35.81%	-22.32%	7.19%	29.74%	-66.14%	20.56%	35.57%	-14.83%	13.45%	10.87%
MM (Constr4):	**SPX**	**ADBE**	**IBM**	**SAP**	**BAC**	**C**	**WFC**	**TRV**	**LUV**	**ALK**	**HA**
MinVar	87.94%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	16.10%	0.00%	0.00%	-4.04%
MaxSharpe	0.00%	50.27%	0.00%	0.00%	0.00%	0.00%	0.00%	17.56%	0.00%	19.52%	12.66%
MM (Constr5):	**SPX**	**ADBE**	**IBM**	**SAP**	**BAC**	**C**	**WFC**	**TRV**	**LUV**	**ALK**	**HA**
MinVar	0.00%	3.53%	34.20%	-1.99%	1.18%	-24.94%	34.79%	52.68%	5.03%	-2.86%	-1.61%
MaxSharpe	0.00%	50.82%	-19.51%	13.81%	36.74%	-76.96%	28.73%	49.95%	-16.54%	18.45%	14.50%

Besides, the results are analyzed based on the following three parts.

4.1. Efficient Frontier

The efficient frontiers under all five constraints are shown in Diagram X. The diagram shows the set of all optimal portfolios that provide the highest expected returns for certain levels of risks

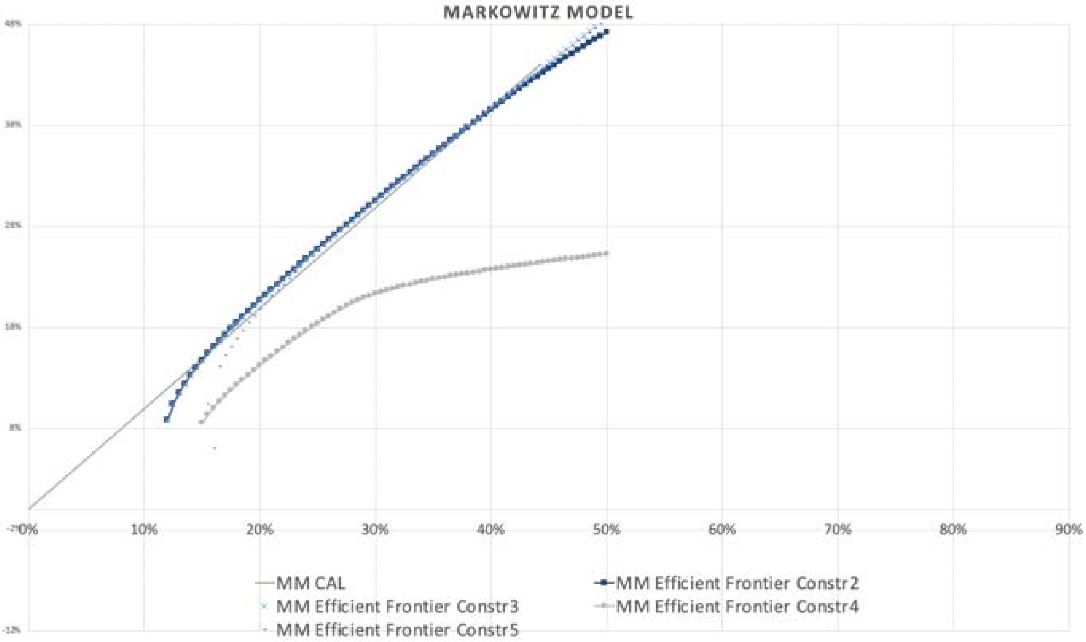

Figure 11. Efficient Frontier under Five Constraints

4.2 Maximum Sharp Ratio

Sharpe ratio is the return yielding capacity for every additional unit of risk taken by it. In this case, I try to find the optimal portfolios that seek to maximize sharp ratios. Table X summarizes the expected returns, standard deviations, and shape ratios of all optimal portfolios under five constraints.

Table 2. Result Table for Max Sharpe

	Return	**StDev**	**Sharpe**
Constraint 1	17.60%	17.70%	0.994
Constraint 2	22.07%	21.33%	1.035
Constraint 3	22.07%	21.33%	1.035
Constraint 4	18.24%	24.75%	0.737
Constraint 5	26.53%	25.98%	1.021

4.3 Minimum Variance

In this case, I try to find the optimal portfolios that seek to minimize volatility (standard deviation). Table X summarizes the expected returns, standard deviations, and shape ratios of all optimal portfolios under five constraints.

Table 3. Result Table for Min Var

	Return	**StDev**	**Sharpe**
Constraint 1	6.72%	11.75%	0.572
Constraint 2	6.97%	11.79%	0.591
Constraint 3	6.69%	11.75%	0.570
Constraint 4	7.01%	14.42%	0.486
Constraint 5	9.38%	15.45%	0.607

5. Conclusion

In this paper, I try to build optimal portfolios through data analysis based on Full Markowitz Model. We first collected past data on ten stocks in the portfolio. Then, we use computation tools from Excel to process the data and find optimization inputs. Finally, we analyze the results under five constraints.

In general, investors should choose the optimal portfolio that satisfies their expectations. The research has provided reliable optimization results. If investors want to maximize expected returns and sharp ratio, they can choose the portfolio "MaxSharpe.” However, they should also tolerance the potential risks involved in investment. If investors want to minimize the risks, they can choose the portfolio “MiniVar.”

However, there are still some limitations in the research. First, the data samples are limited because there are only ten stocks included in the portfolio. Meanwhile, the portfolio only includes three sectors (technology, financial services, and industrials), which lack diversity. Additionally, there are limitations of Full Markowitz Model. For example, the model makes statistical analysis based on past data and fails to include market fluctuations. Also, it excludes any extra cost including taxes or broker commissions. Additionally, there are limitations of the assumptions of the model. The Full Markowitz Model assumes that investors are rational and risk-averse. However, investors may make irrational decisions driven by uncontrolled emotions in real life.

For future research, a few adjustments and improvements will be considered. First, more stocks will be included in the dataset. Also, more sectors will be considered, including healthcare, energy, and consumer defensive. The diversity of stocks provides investors with more options when making decisions and helps to spread the risk between covariances of stocks. Second, more models such as the index model and constant correlation model will be considered. The results will be compared among different models to improve the explanatory power. Finally, all the results are derived from Excel. The computing power based on Excel is limited. If there is another opportunity in the future, software such as Python or Java more is incorporated into the analysis, which will help obtain a better result with efficiency.

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